

Sheet 2

13) Prove $B - A = A^c \cap B$
 إثبات أن $B - A = A^c \cap B$

The conditional probability of an event A given that B has occurred is denoted by $P(A|B) = \frac{P(A \cap B)}{P(B)}$

Sheet 3

- 4) 25 % Failed in math $\rightarrow E_1$
 15 % "chemistry $\rightarrow E_2$
 10 % " both $\rightarrow E_1 \cap E_2$

a) If he failed in chemistry, what is the probability that he failed in math

$$P(E_1|E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)} = \frac{0.1}{0.15}$$

b) If he failed in math, what is the probability that he failed in chemistry

$$P(E_2|E_1) = \frac{P(E_2 \cap E_1)}{P(E_1)} = \frac{0.1}{0.25}$$

c) Failed in math or chemistry

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2) \\ = 0.25 + 0.15 - 0.1 \\ = 0.3$$

5) $P(A) = 0.5$ $P(B) = 0.33$
 $P(A \cap B) = 0.25$

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{0.25}{0.5}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.25}{0.33}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{7}{12}$$

$$P(A^c|B^c) = \frac{P(A^c \cap B^c)}{P(B^c)} = \frac{1 - P(A \cup B)}{1 - P(B)}$$

6) $P(A) = \frac{3}{8}$ $P(B) = \frac{5}{8}$, $P(A \cup B) = \frac{3}{4}$

a) $P(A|B) = \frac{P(A \cap B)}{P(B)}$

b) $P(B|A) = \frac{P(A \cap B)}{P(A)}$

Independent Events:

$$P(B) = P(B|A) = \frac{P(B \cap A)}{P(A)}$$

$$P(A \cap B) = P(A) P(B)$$

1) A, B are independent events

Prove that

a) A^c and B^c are independent

$$P(A^c \cap B^c) = P(A^c) P(B^c) \rightarrow \text{required}$$

$$P(A^c \cap B^c) = 1 - P(A \cup B)$$

$$= 1 - P(A) - P(B) + P(A \cap B)$$

$$= 1 - P(A) - P(B) + \underbrace{P(A) P(B)}_{P(A) P(B)}$$

$$= [1 - P(A)] - P(B) [1 - P(A)]$$

$$= [1 - P(A)] * [1 - P(B)]$$

$$= P(A^c) P(B^c)$$

$\therefore A^c$ and B^c are independent events

b) A and B^c are independent

$$P(A \cap B^c) = P(A) P(B^c) \rightarrow \text{required}$$

2) 3 Fair Coins Find probability that they are all heads if

a) the first coin is head $\rightarrow A$

b) one of coins is head $\rightarrow C$

$$P(A) = 1/8$$

$$P(B) = 1/2 \quad P(C) = 7/8$$

a) $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/8}{1/2} = \frac{2}{8} = \frac{1}{4}$

b) $P(A|C) = \frac{P(A \cap C)}{P(C)} = \frac{1/8}{7/8} = \frac{1}{7}$

10) Probability of a man lives 10 more years is $\frac{1}{4}$, Probability of his wife is $\frac{1}{3}$

a) $E_1 \rightarrow$ man lives 10 more $\rightarrow P(E_1) = \frac{1}{4}$
 $E_2 \rightarrow$ wife " " " $\rightarrow P(E_2) = \frac{1}{3}$

$$P(E_1 \cap E_2) = P(E_1) P(E_2)$$

$$= \frac{1}{4} * \frac{1}{3}$$

Where E_1, E_2 are independent

b) $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$
 $= \frac{1}{4} + \frac{1}{3} - \frac{1}{12}$

c) $P(E_1 \cup E_2)^c = 1 - P(E_1 \cup E_2)$

d) $P(E_2 - E_1) = P(E_2 \cap E_1^c)$

$\Rightarrow P(E_2) - P(E_1 \cap E_2)$ other solution

i) $S = \{bb, bg, gb, gg\}$

$P(A) = 1/2$

$P(B) = 3/4$

$P(A \cap B) = \frac{1}{2}$

$P(A) \cdot P(B) = \frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8}$

$P(A \cap B) \neq P(A)P(B)$

A, B are dependent

Report

3, 8, 12, 13, 17

11) $A \rightarrow$ Family has children from both gender
 $B \rightarrow$ Family has at most one boy

i) Show that A and B are ~~not~~ Independent if Family has 3 child

ii) dependent 2 child

i) Sample Space

$S = \{bbb, bbg, bgb, gbb, bgg, gbg, ggb, ggg\}$

$P(A) = 6/8$

$P(B) = 4/8$

$P(A \cap B) = 3/8$

$P(A) \cdot P(B) = \frac{6}{8} \cdot \frac{4}{8} = \frac{3}{8}$

$\therefore P(A \cap B) = P(A)P(B) = \frac{3}{8}$

$\therefore A, B$ are independent